Carbon Pricing in Input and Export Markets: Multinational Firms and Carbon Leakage using Bernard et al's Model of Global Firms

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Using the model for global firms developed by Bernard et al, this paper presents a way to track carbon dioxide emissions at various points of multinational business and evaluate potential occurrence of carbon leakage when a firm is subjected to regulation in the countries where it operates. The paper develops a framework to analyze firm-level and climate effects of carbon pricing, focusing particularly on imports of intermediate inputs. This is achieved through setting up two policy experiments: carbon pricing in one of the firm's sourcing markets, and retroactive carbon pricing in the export market. This extension of Bernard et al's framework gives chance to think about emissions regulations along multiple margins of international trade. Additionally, an outline of several interesting scenarios for future policy simulations is provided, connecting the research to questions of policy coordination across countries.

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I. Introduction

Regulating and capping the world's carbon emissions has been identified as a shared global goal with the Paris Agreement signed during the COP 21 in 2015, with each country submitting a national pledge, or the Nationally Determined Contribution (NDC). As with many agreements without an imposing authority, regulating CO_2 is a responsibility of each Party to the Agreement.

In a world where increasing amounts of global production are outsourced and trade is globalized, it is important to analyze the impacts of upholding or failing to uphold carbon regulations on multinational firms that operate across national borders and can move their operations or rescale geographically with relative freedom. Scholars and professionals of carbon policy refer to the problem of emissions shifting to unregulated areas when certain relevant areas become regulated as carbon leakage.

Leakage can be approached differently in theoretical and empirical literature. Projections of country or sector welfare after regulation on the border are often done with Computable General Equilibrium models, which aim to create a close replica of an economy with diverse sectors and firms and look at macroeconomic performance. The OECD has produced several broad reaching collaborative working papers, a notable one by Lanzi et al. (2013), which modeled an economy with 17 sectors of different carbon intensity and studied impacts on competitiveness and carbon leakage from different hypothetical linkages. Other types of studies, such as Rocha (2011)**,** offer in-depth studies of impacts of implementing different types of CO_2 emissions regulations domestically, and project the impacts on energy production industry in the US. Under notable empirical work relating to multination corporations and leakage, Dechezleprêtre et al. (2015**)** used self-reported regional emissions reported to the Carbon Disclosure Project with aim to quantify MNCs' internal choice of production location depending on the stringency of its environmental laws, using the EU ETS as the regulated area.

When considering tackling leakage, as well as broader questions of policy coordination which also has space in the Paris Agreement, linking carbon markets comes up often from various perspectives. Some employ economic theory on market efficiency, such as in a detailed review of intuitive economic implications of linking on the countries involved by Flachsland et al (2009). CGE models can also be used for country-level welfare studies from linking, another side of Lanzi et al. (2013). On the more philosophical and simultaneously execution focused end of the spectrum lies works on collaboration theory, diplomacy, and policy design; Aldy et al (2014) and Rodansky et al (2014) and from the Harvard Project on Climate Agreements speculate on how linking could be facilitated so that all parties are better off, countries and firms involved, and leakage minimized, and what measures may make the international community more willing to trust each other's efforts and put more effort into regulation that is meaningful.

Such studies provide insight and deduction, and inform the building of future models. The aim of this paper is to develop a multinational firm piece of one of these simulations based on newest theoretical work in multinational firm theory. This paper includes a description of emissions into on a simplified framework of multinational production and competition developed by Bernard et al (2016). By building onto the extensive presentation of sourcing, production, and export decisions from the original model, this paper contributes to the literature a way of tracking emissions on multiple margins of international trade from the perspective of a multinational firm.

This is achieved through setting up two policy experiments: carbon pricing in one of the firm's sourcing markets, and retroactive carbon pricing in the export market. As Bernard et al extensively described the qualities and conditions of both the sourcing markets and the export destination market, basing this paper on their model allows to examine various forms of carbon pricing enacted on either end, creating realistic and exciting ways to think about movement of global emissions that can be extended to think about policy coordination, such as linking cap and trade markets.

The paper is organized as follows:

The second part sets up the mathematical model and comments on changes from the Bernard paper where necessary. In the following section, two thought experiments that I will be simulating are described, detailing their logic, relevant comparative statics, and solutions for each experiment. Section 3 provides a description of possible extensions into simulating policy coordination in sourcing countries. Finally, section four summarizes the findings of this paper and offers suggestions for further research and relevant policy.

II. Theoretical Framework

The following model of multinational production and sales borrows heavily from Bernard et al (2016), which in turn was built on, especially with respect to mathematical statistics, Eaton and Kortum (2002) and Antràs, Fort, and Tintelnot (2014). This paper simplifies certain parts of their model, and narrows it down to analyze one firm in section 3: policy experiments. The motivation behind this is to put the expansion presented in this paper into focus and define it clearly, so that further studies may integrate the updates into a broader framework. The original model does not solve for a general equilibrium, but provides conditions for choosing arguments optimizing the firm's choice; if the reader is interested in those, they are advised to turn to the later part of the Theoretical Framework section of Bernard et al (2016).

For the rest of the paper, the following subscripts and superscripts are used to refer to variables and parameters on various levels:

- m in reference to the export market in question;
- j to denote a particular sourcing country (when there are multiple, they will be marked as j1, j2…);
- i to denote variables and parameters specific to the production country;
- f to denote variables and parameters specific to the firm (same thing applies – f1, f2, …)
    - Capital F refers to overall "firm level" parameters, such as elasticity of substitution across firms in the market;
- k for product in question
    - Capital K refers to overall "product level" parameters, such as elasticity of substitution across firms in the market;
- g for the sector in which each product k is sold;
- s to mean "sourcing";
- x to mean "export";
- p to mean "production".

II.A. <u>Demand and Consumer Preferences in an Export Market</u>

In Bernard et al, the utility function representing consumer preferences in export market m is:

$$\ln U_m = \int_{g \in \Omega_g} \lambda_{mg} \ln C_{mg} \, dg, \qquad \int_{g \in \Omega_g} \lambda_{mg} dg = 1\,, \tag{1}$$

where λ_{mg} is the relative appeal of sector g to the consumers in market m, and also the share of total national expenditure on that sector; Ω_g refers to the number of different sectors that exist in that market. The consumption index for each sector (C_{mg}) depends on a respective index for each firm in that sector, and that, in turn, a consumption index for all products produced by one firm. The equations for those are provided below.

λ_{mg} refers to the share of spending on sector g in market m,

$$C_{mg} = \left[\sum_{i\in\Omega_i}\sum_{f\in\Omega^F_{mig}}(\lambda_{mif}C_{mif})^{\frac{\sigma^F_g-1}{\sigma^F_g}}\right]^{\frac{\sigma^F_g}{\sigma^F_g-1}}, \qquad \sigma^F_g > 1,\ \lambda_{mif} > 0 \tag{2}$$

σ^F_g is the elasticity of substitution across firms for sector g;

λ_{mif} is a demand shifter, or "firm appeal", that reflects how relatively attractive firm f is to consumers in market m;

Ω^F_{mig} is the set of firms from country i currently selling within sector g in m.

The consumption index for each firm f that supplies its products to market m within sector g is defined over the consumption of each product it sells, k:

$$C_{mif} = \left[\sum_{k\Omega^K_{mif}}(\lambda_{mik}C_{mik})^{\frac{\sigma^K_g-1}{\sigma^K_g}}\right]^{\frac{\sigma^K_g}{\sigma^K_g-1}}, \qquad \sigma^K_g > 1,\ \lambda_{mik} > 0 \tag{3}$$

σ^K_g is the elasticity of substitution across firms for sector g;

λ_{mik} is a demand shifter, or "product appeal", that reflects how relatively attractive product k is to consumers in market m;

Ω^K_{mif} is the set of products that specific firm f from country i is currently selling in country m.

Everywhere else in this paper, where the firm is the only supplier, and they sell only one product to one sector in the economy, all of the "appeal" lambdas are equal to 1; however, having the more general setup makes integrating competition into the model easier to understand.

II.B. <u>Prices in sector g</u>

The price index in sector g in market m is affected by price indices of all participating firms from all production countries, and all products they sell:

$$P_{mg} = \left[\sum_{i\in\Omega_i}\sum_{k\in\Omega^F_{mig}}\left(\frac{P_{mif}}{\lambda_{mif}}\right)^{1-\sigma^F_g}\right]^{\frac{1}{1-\sigma^F_g}}, \tag{4}$$

σ_g^F is the elasticity of substitution across firms within this sector; if above 1, consumers are more sensitive to firms' prices, and overall price level is lower as a result.

$$P_{mif} = \left[\sum_{k\in\Omega_{mik}} \left(\frac{P_{mik}}{\lambda_{mik}}\right)^{1-\sigma_g^K}\right]^{\frac{1}{1-\sigma_g^K}} \tag{5}$$

σ_g^K is the elasticity of substitution across products within this sector; if above 1, consumers are more sensitive to firms' prices. P_{mik} simply refers to the price firm f charges for product k in market m.

Bernard et al also derives the shares of consumer spending on each firm's products, and, further on, on each product – they are specified in (7) and (8). These shares do not play a significant role in this paper and its applications of the model, but are included because "Global Firms" uses them to derive total sales of product k in m, (9).

$$S_{mif} = \frac{\left(\frac{P_{mif}}{\lambda_{mif}}\right)^{1-\sigma_g^F}}{\sum_{i\in\Omega_i}\sum_{k\in\Omega_{mig}^F}\left(\frac{P_{mif}}{\lambda_{mif}}\right)^{1-\sigma_g^F}} = \frac{\partial P_{mg}}{\partial P_{mif}}\frac{P_{mif}}{P_{mg}} \tag{6}$$

$$S_{mik} = \frac{\left(\frac{P_{mik}}{\lambda_{mik}}\right)^{1-\sigma_g^K}}{\sum_{n\in\,\Omega_{mif}}\left(\frac{P_{mik}}{\lambda_{mik}}\right)^{1-\sigma_g^K}} = \frac{\partial P_{mif}}{\partial P_{mik}}\frac{P_{mik}}{P_{mif}} \tag{7}$$

Total sales revenue of product k sold by firm f in market m is:

$$\mathrm{R}_{mik} = (\lambda_{mif})^{\sigma_g^F-1}(\lambda_{mik})^{\sigma_g^K-1}(\lambda_{mg}w_m L_m)(P_{mg})^{\sigma_g^F-1}(P_{mif})^{\sigma_g^K-\sigma_g^F}(P_{mik})^{1-\sigma_g^K}, \tag{8}$$

Where it is assumed that the total expenditure in market m (R_m) is equal to the national income ($w_m L_m$) – where w_m is the wage level in country m, and L_m is the total labor input in m. It is assumed that there is no manufacturing in this economy, that is, that labor is the only input defining production here – something that could be complicated for a more realistic model.

This equation will eventually be used as the firm's revenue function for product k.

II.C. <u>Production Technology and Decisions</u>

a. <u>Final goods production technology for product k</u>

As defined in Bernard et al, all firms produce this good using an array of intermediate inputs l and labor L; each firm has its own productivity factor φ_f which allows for differentiation in production technology.

$$Q_f = \varphi_f \left(\frac{L}{\alpha}\right)^{\alpha} \left(\frac{\int_0^1 Y_k(l)^{\frac{\eta-1}{\eta}} dl}{1-\alpha}\right)^{\frac{(1-\alpha)\eta}{\eta-1}}, \qquad 0 < \alpha < 1, \;\; \eta > 1 \tag{9}$$

φ_f is exogenously given final goods productivity specific to firm f; α is the share of labor in producing product k; η is the elasticity of substitution across each intermediate input. CES is assumed.

The firm uses $Y_k(l)$ amount of inputs l, and those quantities are indexed between 0 and 1; the integral represents summation. Eventually, because the firm uses an array of inputs l, for the math it only matters how much inputs the firm buys in total, and not how much of each. Additionally, it is assumed that all inputs are available in each sourcing country j for which the firm has incurred fixed sourcing costs, and that the distribution of average prices for each input is shaped the same.[1]

b. Intermediate input productivity and prices

Bernard et al uses the Fréchet distribution to model fluctuations in intermediate input productivity and prices.[2] The key upsides of using this cumulative distribution function are its being "s-shaped", which shows that for both productivity and prices, the extremes are less likely than the average, and the property that the minimum values from many sets of these distributions are also distributed in the same way, which becomes important.

The cost of sourcing an intermediate input l from j1 is its price as experienced by the firm. This price depends on wage level in country j1, w_{j1}, sourcing costs from j1 to the production location, d_{j1}^{s}, and intermediate input productivity in that country, $z_{j1}(l)$.

$$a_{j1}(l) = \frac{w_{j1} d_{j1}^{s}}{z_{j1}(l)} \tag{10}$$

Input productivity fluctuations in country j1 are modeled with a Fréchet distribution:

$$G_{j1}(z) = e^{-T_{j1} z^{-\theta}} \tag{11}$$

Where G(z) means Prob (Z<z) – probability of the drawn value being less than a given value of z; T_{j1} is the Fréchet scale parameter indicating the average productivity of

[1] Eaton and Kortum (2002) have developed this pricing method, and Bernard et al helpfully refers to them for a broader explanation.
[2] Again, Eaton and Kortum (2002), who developed this input pricing method, have a more detailed explanation of the features and use of this distribution.

intermediate inputs from country j1, and θ is the Fréchet shape parameter that determines the dispersion of that productivity.[3]

Bernard et al assumes the average input productivity and the dispersion of productivity is the same for all inputs sold in country j, but lets z be drawn for each input, so input prices vary slightly for each of the inputs.

For each country j that the firm buys inputs from, the distribution of prices for intermediate inputs is:

$$G_j\left(a, \Omega_{f1}^s\right) = 1 - e^{-T_{j1}\left(w_{j1}d_{j1}^s\right)^{-\theta} a^\theta}, \qquad j \in \Omega_{f1}^s \quad ^4 \tag{12}$$

Bernard et al and the papers they base their model on justify using the Fréchet distribution because its mathematical properties give that the minimum of Fréchet distributed variables is itself Fréchet distributed. The probability of minimum price A<a for firm f across all of its sourcing countries is:

$$G_f\left(a, \Omega_{f1}^s\right) = 1 - e^{-T_{j1}\left(w_{j1}d_{j1}^s\right)^{-\theta} a^\theta} * e^{-T_{j2}\left(w_{j2}d_{j2}^s\right)^{-\theta} a^\theta} * \dots * e^{-T_{jn}\left(w_{jn}d_{jn}^s\right)^{-\theta} a^\theta}$$

$$= 1 - e^{-\sum_{j\in\Omega_j^s} T_j\left(w_j d_j^s\right)^{-\theta} a^\theta} \tag{13}$$

$$G_f\left(a, \Omega_f^s\right) = 1 - e^{-\Phi_{f1}\left(\Omega_f^s\right)a^\theta}, \quad \text{where } \Phi_f\left(\Omega_f^s\right) \equiv \sum_{j\in\Omega_j^s} T_j\left(w_j d_j^s\right)^{-\theta} \tag{14}$$

Given this distribution for minimum prices, the probability of firm f buying an intermediate input for product k from country j is:

$$\rho_j\left(\Omega_j^s\right) = \frac{T_j\left(w_j d_j^s\right)^{-\theta}}{\sum_{j\in\Omega_j^s} T_j\left(w_j d_j^s\right)^{-\theta}} = \frac{T_j\left(w_j d_j^s\right)^{-\theta}}{\Phi_f\left(\Omega_f^s\right)} \tag{15}$$

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[3] As explained in Eaton and Kortum (2002): the higher theta is, the more likely it is on any random draw that productivity is within a close proximity of the average; the lower it is, the more varied the productivity is in that country.

[4] Most of the formula is straightforward to derive: the distribution gets flipped from the distribution of z (that is, if input productivity is higher, by the formula of how its price a is determined gives that the price will be lower). The above formula simply plugs in $z = \frac{w_{j1}d_{j1}^s}{a_{j1}(l)} = w_{j1}d_{j1}^s * a^{-1}$ and makes

$$G_{j1}(z) = e^{-T_{j1}(w_{j1}d_{j1}^s a^{-1})^{-\theta}} = e^{-T_{j1}(w_{j1}d_{j1}^s)^{-\theta}(a^{-1})^{-\theta}} = e^{-T_{j1}(w_{j1}d_{j1}^s)^{-\theta}a^\theta};$$

The $\Phi_f(\Omega_f^s)$ is important to note and define because of the it plays a role in further calculations. Bernard et al defines it as the *firm supplier access* that describes how good the firm's access to cheaper inputs is. With each new sourcing country for which the firm incurs fixed sourcing costs, this supplier access measure increases. As the equations below show, and as explored in the next section, having access to more sourcing markets decreases the firm's variable unit costs.

II.D. <u>Firm's Cost Functions</u>

Variable unit costs dual to final goods production technology:

$$v_{fk}(\varphi, \Omega_{sf}) = \frac{1}{\varphi_{f1}} w_i{}^{\alpha} \left[\int_0^1 a_j(l)^{1-\eta_g}\, dl\right]^{\frac{1-\alpha}{1-\eta_g}} \tag{16}$$

[5]

Plugging in the distribution for minimum intermediate input prices, variable unit costs can be expressed as:

$$v_{fk}(\varphi, \Omega_j^s) = \frac{1}{\varphi_{f1}} w_i{}^{\alpha} (\gamma_k)^{1-\alpha} \left[\Phi_{f1k}(\Omega_{f1}^s)\right]^{-\frac{1-\alpha}{\theta_k}} \tag{17}$$

where $\gamma_k = \left[\Gamma\left(\frac{\theta_k + 1 - \eta_g}{\theta_k}\right)\right]^{\frac{1}{1-\eta_g}}$.

The Gamma function is used widely in statistics; this expression requires $\theta_k > \eta_g - 1$.[6]

The firm's total cost function is the sum of total variable costs and all fixed sourcing costs incurred during production:

$$TC(\varphi, \Omega_j^s, Q_{ik}) = \frac{w_i{}^{\alpha} (\gamma_k)^{1-\alpha} \left[\Phi_{f1k}(\Omega_{f1}^s)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}} Q_{ik} + \sum_{j \in \Omega_f^s} w_i F_{ij}^S \tag{18}$$

Note that this is purely production costs; when marginal costs are incorporated into the profit equation later on, the variable part also incorporates export costs, which are modeled as iceberg trade costs ($d_{mi}^x > 1$ and multiplies the whole variable cost part). The profit maximization problem later on also includes fixed production costs.

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[5] The integral represents summation, and it is assumed that there is a constant elasticity of substitution between all inputs l, like elsewhere, which is represented by the exponents

[6] Remember that $\eta_g > 1$ to represent that the inputs are elastic in the production process; this ensures that the argument of the Gamma function is positive, because Gamma functions are only defined for positive arguments

II.E. Sales and Profits

The firm's profit maximization problem:

$$\max_{\{P_{mik}:m\in\Omega_f^x,k\in\Omega_{mif}\}} \Pi_{igf} = \begin{Bmatrix} \sum_{m\in\Omega_f^x}\sum_{k\in\Omega_{mif}} \mathrm{P}_{mik}\mathrm{Q}_{mik}\left(\mathrm{P}_{mik}\right) - \frac{(d_{mi}^x)w_i{}^{\alpha}(\gamma_k)^{1-\alpha}\left[\Phi_{f1k}\left(\Omega_{f1}^s\right)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}} \mathrm{Q}_{mik}(\mathrm{P}_{mik}) \\ -\sum_{m\in\Omega_f^x}\sum_{k\in\Omega_{mif}} w_i F_{mik}^k - \sum_{m\in\Omega_f^x} w_i F_{mi}^x - \sum_{j\in\Omega_{jf}^s} w_i F_j^s - w_i F_f^p \end{Bmatrix} \quad (19)$$

Derived in Bernard et al, the equilibrium pricing rule is:

$$P_{mik} = \mu_{mif} \frac{(d_{mi}^x)w_i{}^{\alpha}(\gamma_k)^{1-\alpha}\left[\Phi_{f1k}\left(\Omega_{f1}^s\right)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}} \quad (20)$$

Bernard et al allows firms to use variable firm specific markups, μ_{mif}, which depend on perceived elasticity of demand for the firm's products in the export market. Since the purpose of this paper is to integrate emissions rather than analyze pricing and competition dynamics in the export market (they are provided more for completeness of policy experiments), the assumption throughout is that markups are stationary. In a broader simulation interested in competition, this assumption can of course be relaxed.

Profits from sector g for firm f are:

$$\Pi_{igf} = \sum_{m\in\Omega_f^x}\sum_{k\in\Omega_{mif}} \left(\frac{\mu_{mif}-1}{\mu_{mif}}\right)\mathrm{R}_{mik} - \sum_{m\in\Omega_f^x}\sum_{k\in\Omega_{mif}} w_i F_{mik}^k - \sum_{m\in\Omega_f^x} w_i F_{mi}^x - \sum_{j\in\Omega_{jf}^s} w_i F_j^s - w_i F_f^p \quad (21)$$

Following Bernard et al's assumption that the firm incurs constant marginal costs to recover variable costs from sales, represented by $\frac{R_{mik}}{\mu_{mif}}$, the level of intermediate inputs imported by firm f for product k from each source country j are a function of those costs and the sourcing probability:

$$M_j = \frac{T_j\left(w_j d_j^s\right)^{-\theta}}{\sum_{j\in\Omega_j^s} T_j\left(w_j d_j^s\right)^{-\theta}} \left[\sum_{m\in\Omega_f^x} \frac{R_{mik}}{\mu_{mif}}\right] = \rho_j\left(\Omega_j^s\right)\left[\sum_{m\in\Omega_f^x} \frac{R_{mik}}{\mu_{mif}}\right] \quad (24)$$

Finally, sales of product k can now be expressed by plugging in the equilibrium pricing rule from (20) into the sales equation from (8):

$$\mathrm{R}_{mik} = \left(\lambda_{mif}\right)^{\sigma_g^F-1}(\lambda_{mik})^{\sigma_g^K-1}\left(\lambda_{mg} w_m L_m\right)\left(P_{mg}\right)^{\sigma_g^F-1}\left(P_{mif}\right)^{\sigma_g^K-\sigma_g^F}\left(\mu_{mif}\frac{(d_{mi}^{x}) {w_i}^{\alpha}(\gamma_k)^{1-\alpha}\left[\Phi_{f1k}\left(\Omega_{f1}^{s}\right)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}}\right)^{1-\sigma_g^K} \tag{25}$$

II.F. Emissions

The original contribution of this paper is integrating a way to model total emission count from the sourcing process into the firm's production decisions and variables. An equation expressing total emissions from sourcing in country j relates the total quantity of inputs firm f buys from that country and the technology that country j uses to produce those inputs.

As per Bernard et al., it is assumed that input markets are perfectly competitive and have access to the same average input productivity described above; in this paper, that average productivity is not related to the emissions generated by state of the art technology for producing each input l in each country j, but it is assumed that all input firms in country j use the same technology and thus generate the same amount of emissions from producing all quantities of l. This assumption makes sense in talking about certain inputs, such as natural resources: oil or precious metals from different regions may have the same productivity when used to produce final goods, but certain countries may have more environmentally friendly extraction processes.

This version of emissions modeling makes two assumptions to simplify the math. Firstly, each of the intermediate inputs produces the same amount of $CO2$, thus the same equation can be applied to the sum of inputs to calculate total emissions. This is not a very realistic assumption, but one that can be relaxed in future research. This simplification is valid to make because the elasticity of substitution of inputs is fixed by the sector in the short term: even if thinking about emissions, the firms still would not think about buying the inputs in different proportions.

This paper also assumes that when carbon regulation is imposed in each sourcing market, it affects all sectors that each input l belongs to uniformly – thus increases the sourcing cost for each input by the same amount; the original Bernard et al. paper already employed the assumption that sourcing costs are uniform across inputs. A more realistic way to think about it could be understanding sourcing cost for each input as an average sourcing cost, or to expand the math to use alternative specifications in the future.

When thinking about a carbon policy in the export market, no further assumptions are necessary – for one, the model assumes iceberg trade costs, and further – there is only one product being sold in most experiments carried out in this paper. It is possible to have the export market impose different levels of carbon taxation on different products the firm sells in m, which in turn would translate into different changes in exporting costs for each product, but it is not the focus of this paper.

With the above assumptions clarified, the simplest way to model emissions from each sourcing country j is with a linear equation relating the amount of inputs l that the firm buys from that country to a coefficient of emissions intensity in production. Given that the amount of inputs the firm buys from j1 is the same as the level of imports from j1, emissions from country j1 are:

$$E_{j1} = C_{j1} * M_{j1} \tag{26}$$

Allowing emissions intensity parameters to vary across all countries j in the firm's set for sourcing countries, total emissions by firm 1 can be expressed as:

$$E_f = \begin{bmatrix} M_{j1} & M_{j2} & \dots & M_{jn} \end{bmatrix} * \begin{bmatrix} C_1 \\ C_2 \\ \dots \\ Cn \end{bmatrix} = \sum_{j \in \Omega_j^s} C_j * M_j \tag{27}$$

The next section makes use of this model, narrowing everything down to one firm, and walks through two different carbon pricing policy experiments.

III. Policy experiments within given set of countries

This section uses the model set up in the previous section to examine various carbon pricing policy scenarios. The initial bare bones setup common to these scenarios is one firm f exporting one product k to one export market m, and sourcing 3 intermediate inputs l from 3 source countries indexed by j1, j2. j3. This section aims to see the changes in the location distribution for imports, total emissions levels, and firm sales and profits spurred on by the policy changes.

I. <u>Carbon pricing scheme in a sourcing country</u>

A policy of regulating carbon emissions by imposing a price on them, either by a tax or an emissions trading scheme, will raise the costs for all intermediate input producers. In turn, they will transfer those cost increases at least partially onto the firms buying their products - input prices in the newly regulated country are going to rise. From the way input prices are determined, this price increase will be seen as an increase in sourcing costs $d_{j1}^{s}$from country j1 by t_{j1}^{i}, the total amount effectively paid in carbon taxes to country j1.

Thus, now:

$$d_{j1}^{s}{}' = d_{j1}^{s} + t_{j1}^{i} \tag{28}$$

and

$$a_{j1}\,(l) = \frac{w_{j1}*(d_{j1}^{s}+t_{j1}^{i})}{z_{j1}(l)}, \tag{29}$$

where

$$t_{j1}^{i} = c * E_{j1}; \tag{30}$$

Thus, logically:

$$\frac{\partial d_{j1}^{s}{}'}{\partial E_{j1}} = \frac{\partial d_{j1}^{s}{}'}{\partial t_{j1}^{i}} * \frac{dt_{j1}^{i}}{\partial E_{j1}} = c: \tag{31}$$

total amount paid to country j1 in carbon taxes is c, price for emitting a unit of carbon, multiplied by total carbon emissions in j1.

This policy affects the firm's variable costs, which is reflected through the firm supplier access parameter, $\Phi_{f1}(\Omega_{f1}^{s})$:

$$v_{fk}(\varphi, \Omega_{j}^{s}) = \frac{1}{\varphi_{f1}} w_{i}{}^{\alpha}(\gamma_{k})^{1-\alpha}\left[\Phi_{f1}(\Omega_{j}^{s})\right]^{-\frac{1-\alpha}{\theta_{k}}} \tag{16}$$

$$\frac{\partial v_{fk}}{\partial \Phi_{f1}} = -\left(\frac{1-\alpha}{\theta_{k}}\right)\frac{1}{\varphi_{f1}} w_{i}{}^{\alpha}(\gamma_{k})^{1-\alpha}\left[\Phi_{f1}(\Omega_{j}^{s})\right]^{-\frac{1-\alpha+\theta_{k}}{\theta_{k}}} < 0 \tag{32}$$

$$\frac{\partial \Phi_{f1}}{\partial d_{j1}^{s}} = -\theta\, T_{j1}\left(w_{j1} d_{j1}^{s}\right)^{-\theta-1} w_{j1} \quad = \quad \frac{-\theta\, T_{j1} w_{j1}}{\left(w_{j1} d_{j1}^{s}\right)^{\theta+1}} \quad < 0 \tag{33}$$

and

$$\frac{\partial \Phi_{f1}}{\partial E_{j1}} = \frac{\partial \Phi_{f1}}{\partial d_{j1}^{s}}\frac{\partial d_{j1}^{s}}{\partial E_{j1}} = \quad \frac{-c\theta\, T_{j1} w_{j1}}{\left(w_{j1} d_{j1}^{s}\right)^{\theta+1}} \tag{34}$$

Higher sourcing costs decrease firm supplier access, and this decrease is inversely proportional to the firm's emissions by the factor of the carbon price set in j1.

From that, we see that

$$\frac{\partial v_{fk}}{\partial d_{j1}^{S}} > 0 \qquad \text{and} \qquad \frac{\partial v_{fk}}{\partial E_{j1}^{1}} > 0 \tag{35}$$

Proposition 1. If one of the firm's sourcing countries starts pricing carbon emissions, that leads to an increase in the firm's variable production costs.

Increased sourcing costs also affect the probability that the country in which the policy is implemented has the lowest input prices for that period – thus, that firm f would choose to buy from there. From (16), that probability changes:

$$\frac{\partial \rho_{j1}}{\partial d_{j1}^{S}} = \frac{\partial \left(\frac{T_{j1}\left(w_{j1} d_{j1}^{S}\right)^{-\theta}}{\Phi_f} \right)}{\partial d_{j1}^{S}} = \frac{\left[\frac{-\theta\, T_{j1} w_{j1}}{\left(w_{j1} d_{j1}^{S}\right)^{\theta+1}}\right] \Phi_f - \left[T_{j1}\left(w_{j1} d_{j1}^{S}\right)^{-\theta} \right] \left[\frac{-\theta\, T_{j1} w_{j1}}{\left(w_{j1} d_{j1}^{S}\right)^{\theta+1}}\right]}{\left[\Phi_f\right]^2} \tag{36}$$

$$= -\frac{\sum_{j \neq j1} T_j\left(w_j d_j^{S}\right)^{-\theta}}{\left[\Phi_f\right]^2} < 0$$

Intuitively, the probability that the firm buys inputs from j1 decreases. If we multiplied the above result by $\frac{\partial d_{j1}^{S}}{\partial E_{j1}} = c$, we would see that the more this firm emits in country j1, the greater will be the substitution away from buying inputs from there in the future.

Proposition 2. If one of the firm's sourcing countries starts pricing carbon emissions, that leads to a substitution away from that country for buying intermediate inputs; the strength of the substitution effect is proportional to how much total emissions the firm causes in that country.

If we assume the firm uses constant markups over production and export costs, then the price the firm charges for product k increases:

$$\frac{\partial P_{mik}}{\partial d_{j1}^{S}} = \frac{\partial P_{mik}}{\partial v_{fk}} \frac{\partial v_{fk}}{\partial \Phi_{f1}} \frac{\partial \Phi_{f1}}{\partial d_{j1}^{S}} = (+)(-)(-) \quad > 0 \tag{37}$$

Given the demand in market m, we can look into how the sales of product k change using (29). Note that $\sigma_g^K > \sigma_g^F > 1$[7] and that, given that firm f is a the only firm in the market, $P_{mif} = P_{mg} = P_{mik}$ from (5) and (6); the appendix details the math.

[7] Bernard et al assume that substitution across firms and products is elastic, that is the reason for exponents > 1; substitution across products being more elastic than substitution across firms refers

Using these conditions, we get no revenue effect of the taxes – price increases do not affect revenue:

$$\frac{\partial R_{mik}}{\partial d_{j1}^{s}} = \frac{\partial R_{mik}}{\partial P_{mik}} \frac{\partial P_{mik}}{\partial d_{j1}^{s}} = 0 * (+) = 0 \tag{38}$$

Directly following that, givent that the firm's profits are:

$$\Pi_{igf} = \left(\frac{\mu_{mif-1}}{\mu_{mif}}\right) \mathrm{R}_{mik} - w_i F_{mik}^{k} - w_i F_{mi}^{x} - \sum_{j \in \Omega_{jf}^{s}} w_i F_j^{s} - w_i F_f^{p}; \tag{39}$$

and given that we know that $\frac{\partial R_{mik}}{\partial d_{j1}^{s}} = 0$, if markups are stationary, $\frac{\partial \Pi_{igf}}{\partial d_{j1}^{s}} = 0$. (40)

Using this result, and change in sourcing probability from (37), we can see that total imports from j1 defined in (28) decrease:

$$\frac{\partial M_{j1}}{\partial d_{j1}^{s}} = \frac{\partial \rho_{j1}}{\partial d_{j1}^{s}} \left[\frac{R_{mik}}{\mu_{mif}}\right] + \frac{\partial \left[\frac{R_{mik}}{\mu_{mif}}\right]}{\partial d_{j1}^{s}} \rho_{j1} \quad = \quad (-) + 0 \quad < 0 \tag{41}$$

Imports from the other two countries relate to increased sourcing costs in j1 through their sourcing probability as well, which is determined by the change in firm supplier access. Very similarly, we know that

$$\frac{\partial M_{j2}}{\partial d_{j1}^{s}} = \frac{\partial \rho_{j2}}{\partial d_{j1}^{s}} \left[\frac{R_{mik}}{\mu_{mif}}\right] + \frac{\partial \left[\frac{R_{mik}}{\mu_{mif}}\right]}{\partial d_{j1}^{s}} \rho_{j2}; \tag{42}$$

And we can find out that the math confirms the intuitive result:

$$\frac{\partial \rho_{j2}}{\partial d_{j1}^{s}} = \frac{\partial \left(\frac{T_{j2}\left(w_{j2} d_{j2}^{s}\right)^{-\theta}}{\Phi_f}\right)}{\partial d_{j1}^{s}} = -\left(\Phi_f\right)^{-2} T_{j2}\left(w_{j2} d_{j2}^{s}\right)^{-\theta} \frac{\partial \Phi_{f1}}{\partial d_{j1}^{s}} \tag{43}$$

$$= \quad (-)\,(-) \quad > 0$$

With a starting assumption that average input productivity, wages and sourcing costs in all three countries are the same, the increase in the probability should be identical for both j2 and j3, and half of the decrease in the probability of sourcing from j1.

to the "cannibalization effect" - that is, by introducing a new product, the firm hurts the sales of other products it is selling.

Plugging in the above results to (41), we see that $\frac{\partial M_{j2}}{\partial d_{j1}^s} > 0$ and $\frac{\partial M_{j3}}{\partial d_{j1}^s} > 0$ (44)

Proposition 3. A carbon policy in j1 will cause imports from j1 to decrease and imports from j2, j3 to increase. More generally, these shifts will be proportional to average intermediate input productivity in each country, T_j, and inversely proportional to wage and sourcing cost levels from those countries, $w_j,\ d_j^s$, given how those values show up in the formulas for $\frac{\partial \rho_{j2}}{\partial d_{j1}^s}$ and $\frac{\partial \rho_{j3}}{\partial d_{j1}^s}$.

Now, after seeing that the firm does reshuffle imports, it is possible to write out how that affects the firm's total emissions. First, logically, since

$$E_f = \sum_{j \in \Omega_j^s} C_j M_j \text{, we know that } \frac{\partial E_{j1}}{\partial d_{j1}^s} = C_1 \frac{\partial M_{j1}}{\partial d_{j1}^s} < 0 \quad (45)$$

After having derived how the import quantities from all countries change given the increase in sourcing costs from j1, we can express the total change in emissions by the firm as:

$$\frac{\partial E_f}{\partial d_{j1}^s} = C_1 \frac{\partial M_{j1}}{\partial d_{j1}^s} + C_2 \frac{\partial M_{j2}}{\partial d_{j1}^s} + C_3 \frac{\partial M_{j3}}{\partial d_{j1}^s} = (-) + (+) + (+) \lesseqgtr 0 \quad (46)$$

Proposition 4. The change in total emissions depends on how carbon intensive the process of intermediate input production is in each country (expressed by C_j) and on the differences in characteristics of that production in the countries with no policy, such as average intermediate input productivity, wages, and sourcing costs.

Very similarly, extensions can be made to model regulations in multiple sourcing countries j.

II. Retroactive carbon pricing in the export market

Measures like a border adjustment tax (BAT) or another type of regulation that an export market can impose to pass carbon costs onto the firms who want to sell to its consumers can be expressed through an increase in export costs that the firm incurs, d_{mi}^x. As other trade costs, they are modeled as iceberg trade costs in the Bernard et al paper; this transforms the marginal unit costs (as incorporated into the price):

$$P_{mik} = \mu_{mif} \frac{(d_{mi}^{x}+t_{m}^{i}) w_i{}^{\alpha} (\gamma_k)^{1-\alpha} \left[\Phi_{f1k}\left(\Omega_{f1}^{s}\right)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}} \tag{47}$$

But the increase in the export costs has to be proportional to the emissions level that the firm is responsible for overall. This relationship is inverse: the larger the firm's measured (or estimated) emissions during the production process, the smaller adjustment the country where final goods are sold will aim to impose. Like in the case of carbon pricing in a sourcing market, the simplest way to show this relationship is through a linear equation:

$$t_m^i = b * E_f \tag{48}$$

where b is some form of a per unit carbon tax.

In a similar way as in the previous section, we see that the increase in the export costs will raise the firm's price for product k by an amount proportional to how much it is emitting:

$$\frac{\partial P_{mik}}{\partial E_f} = \frac{\partial P_{mik}}{\partial d_m^{x}{}'} \frac{\partial d_m^{x}{}'}{\partial t_m^i} \frac{\partial t_m^i}{\partial E_f} = \frac{\mu_{mif}\, w_i{}^{\alpha} (\gamma_k)^{1-\alpha} \left[\Phi_{f1k}\left(\Omega_{f1}^{s}\right)\right]^{-\frac{1-\alpha}{\theta_k}}}{\varphi_{f1}} * b \tag{49}$$

Proposition 1. Taxing carbon emissions retroactively will raise the firm's price for product k by an amount proportional to how much it is emitting.

Although, as the previous section made clear, in the case of this firm being a monopoly, price increases do not actually decrease sales or profits, each unit of emissions is costing the firm a portion of its variable costs, it still has an incentive to minimize its total emissions subject to some maximum amount of tax it wants to pay, T_f:

$$\min_{\{Mj1,Mj2,Mj3\}} E_f = C_{j1} M_{j1} + C_{j2} M_{j2} + C_{j3} M_{j3} \tag{50}$$

Or, really,

$$\min_{\{\rho j1,\, \rho j2 \rho j3\}} E_f = \left[\frac{R_{mik}}{\mu_{mif}}\right] \left[\, C_{j1}\rho_{j1} + C_{j2}\rho_{j3} + C_{j3}\rho_{j3}\right] \tag{51}$$

The main takeaway from Cramer's rule calculations (detailed in the Appendix) are:

$$\frac{\partial \rho_j^*}{\partial C_j} \neq 0 \quad \text{and} \; \frac{\partial \rho_j^*}{\partial C_j} = f(\rho_{j1}, \rho_{j2}, \rho_{j3}, C_{j1}\, C_{j2},\; C_{j3}) \qquad (52)$$

if the emission intensities are allowed to change; otherwise, without a constraint, the firm will assume that it is currently at the minimum, and not change the sourcing probabilities or import levels, but rather pay the higher costs. It would be possible to model this as a constrained optimization problem as well, with the firm setting the cap for ideal case emissions.

Proposition 2. When the export market imposes a retroactive carbon pricing policy, the changes in sourcing probabilities from the sourcing countries will depend on the qualities of those sourcing markets (reflected by sourcing probabilities) and on the emissions intensity in those markets, if the emission intensities change at all; otherwise, sourcing probabilities will not change.

Given the above result on sourcing probabilities, it follows that

$\frac{\partial M_j^*}{\partial C_j} \neq 0$ and $\frac{\partial M_j^*}{\partial C_j} = f(\rho_{j1}, \rho_{j2}, \rho_{j3}, C_{j1}\, C_{j2},\; C_{j3})$ as well under the same conditions. (53)

Proposition 3. When the export market imposes a retroactive carbon pricing policy, the changes in import levels from the sourcing countries will depend on the qualities of those sourcing markets (reflected by sourcing probabilities) and on the emissions intensity in those markets if the emission intensities change at all; otherwise, import levels will not change.

IV. Extensions

The sections before this part have set up ample space to use the model to simulate policy changes of various sizes and types, including coordination between countries.

Interesting experiments:

- Without current emissions intensities in the sourcing countries changing, how would imports of intermediate inputs be redistributed if all countries imposed different carbon prices? How would the size of those redistributions be affected by changes in price size when emissions intensities do change?
    - Allows to extend the simpler model into a broader comparison;
    - Allows for sourcing countries to change their state of the art production technology to a cleaner one, either due to the policy or an unrelated factor;

- How would coordination to introduce a uniform carbon price among source countries affect the firm?

- How would all of the above change if competition was introduced into market m?

- How would all of the above change if firms competed in several export markets? In sectors with various differences?

Overall, it is possible to build a firm-focused CGE model with enough breadth to cover some or all of these scenarios with the work done in the previous sections.

V. Discussion and Conclusions

Using the model for global firms developed by Bernard et al, this paper presents a way to think about emissions at various points of multinational business. Pricing carbon on both ends work through different channels, but eventually integrates the production and emissions intensity characteristics of sourcing countries into the firm's optimal choices and shows that those choices are interdependent. This model gives chance to think about emissions regulations along multiple margins of international involvement, focusing in particular on imports of intermediate inputs when the firm is subjected to regulation.

This model could also be used to model companies already based in country m, which has been heretofore described as the export market – they incur very similar costs if they source and produce elsewhere, and could be subject to both sourcing costs and a form of import tax or carbon regulation upon importing the final products back home.

Future extensions are welcome and encouraged, especially ones described in the previous section. A large part of the firm's total carbon emissions may happen during the final good production process, which is overlooked in this model. Additionally, a natural extension to simulate a more realistic situation in the export market would be to add more firms to the model to introduce competition and variable markups, which is straightforward to do using Bernard et al's model. In that situation, there are multiple potentially interesting impact channels to explore: not overlapping sourcing countries, change in "firm appeal" parameters due to consumer preferences for greener production processes, various pricing games by firms, etc.

Additionally, future work may introduce complexity in modeling emissions from the sourcing process, relaxing the assumption that all inputs are affected uniformly. Intermediate inputs may in practice belong to different sectors, and it is possible that only some of them would fall under regulation. A broader model may allow and specify substitution between particular inputs, causing leakage in the form of switching inputs altogether, not just the locations from which they are purchased.

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Appendix

Deriving (38):

$\sigma_g^K > \sigma_g^F > 1$

and

$$P_{mif} = \left(\frac{P_{mik}}{\lambda_{mik}}\right)^{\frac{1-\sigma_g^K}{1-\sigma_g^K}} = \frac{P_{mik}}{\lambda_{mik}} = P_{mik}\ ;\ P_{mg} = \left(\frac{P_{mif}}{\lambda_{mif}}\right)^{\frac{1-\sigma_g^F}{1-\sigma_g^F}} \frac{P_{mik}}{\lambda_{mif}\lambda_{mik}} = P_{mik} \text{ from (5) and (6).}$$

$$\frac{\partial R_{mik}}{\partial d_{j1}^s} = \frac{\partial P_{mik}}{\partial d_{j1}^s}\frac{\partial R_{mik}}{\partial P_{mik}}$$

$$= \left[\frac{\partial P_{mik}}{\partial d_{j1}^s}\right] \frac{\partial}{\partial P_{mik}} \left[\left(\lambda_{mif}\right)^{\sigma_g^F-1}(\lambda_{mik})^{\sigma_g^K-1}\left(\lambda_{mg} w_m L_m\right)\left(P_{mg}\right)^{\sigma_g^F-1}\left(P_{mif}\right)^{\sigma_g^K-\sigma_g^F}(P_{mik})^{1-\sigma_g^K}\right]$$

$$= \left[\frac{\partial P_{mik}}{\partial d_{j1}^s}\right] \frac{\partial}{\partial P_{mik}} \left[(1)^{\sigma_g^F-1}\,(1)^{\sigma_g^K-1}\,(w_m L_m)(P_{mik})^{\sigma_g^F-1}(P_{mik})^{\sigma_g^K-\sigma_g^F}(P_{mik})^{1-\sigma_g^K}\right]$$

$$= \left[\frac{\partial P_{mik}}{\partial d_{j1}^s}\right] \frac{\partial}{\partial P_{mik}} \left[(1)(w_m L_m)(P_{mik})^{\sigma_g^F - 1 + \sigma_g^K - \sigma_g^F + 1 - \sigma_g^K}\right]$$

$$= \left[\frac{\partial P_{mik}}{\partial d_{j1}^s}\right] \frac{\partial}{\partial P_{mik}} \left[(w_m L_m)(P_{mik})^0\right] = \left[\frac{\partial P_{mik}}{\partial d_{j1}^s}\right] \frac{\partial}{\partial P_{mik}} \left[(w_m L_m)\right] = (+)\,(0) = 0$$

Using these conditions, we get no revenue effect of the taxes – price increases do not affect revenue:

$$\frac{\partial R_{mik}}{\partial d_{j1}^s} = 0 \qquad (38)$$

Deriving comparative statics for (51):

$$\min_{\{Mj1,\,Mj2,Mj3\}} E_f = \left[\frac{R_{mik}}{\mu_{mif}}\right] \left[\, C_{j1}\rho_{j1} + C_{j2}\rho_{j2} + C_{j3}\rho_{j3}\right]$$

FOC:

1. $$\frac{\partial E_f}{\partial \rho_{j1}} = C_{j1} + C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j1}} + C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j1}} = 0$$

2. $$\frac{\partial E_f}{\partial \rho_{j2}} = C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j2}} + C_{j2} + C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j2}} = 0$$

3. $$\frac{\partial E_f}{\partial \rho_{j3}} = C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j3}} + C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j3}} + C_{j3} = 0$$

After taking the total differentials of the FOCs, the Hessian is:

$$\underline{\underline{H}} = \begin{bmatrix} C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j1}{}^2} + C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j1}{}^2} & C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j1}\partial \rho_{j2}} & C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j1}\partial \rho_{j3}} \\ C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j1}\partial \rho_{j2}} & C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j2}{}^2} + C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j2}{}^2} & C_{j3}\frac{\partial \rho_{j3}}{\partial \rho_{j3}\partial \rho_{j2}} \\ C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j1}\partial \rho_{j3}} & C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j3}\partial \rho_{j2}} & C_{j1}\frac{\partial \rho_{j1}}{\partial \rho_{j3}{}^2} + C_{j2}\frac{\partial \rho_{j2}}{\partial \rho_{j3}{}^2} \end{bmatrix}$$

And

$$\underline{\underline{H}} \begin{bmatrix} \partial \rho_{j1} \\ \partial \rho_{j2} \\ \partial \rho_{j3} \end{bmatrix} = \begin{bmatrix} -dC_1 - \frac{\partial \rho_{j2}}{\partial \rho_{j1}}\, dC_2 - \frac{\partial \rho_{j3}}{\partial \rho_{j1}}\, dC_3 \\ -dC_2 - \frac{\partial \rho_{j1}}{\partial \rho_{j2}}\, dC_1 - \frac{\partial \rho_{j3}}{\partial \rho_{j2}}\, dC_3 \\ -dC_3 - \frac{\partial \rho_{j1}}{\partial \rho_{j3}}\, dC_1 - \frac{\partial \rho_{j2}}{\partial \rho_{j3}}\, dC_2 \end{bmatrix}$$

Using Cramer's rule:

$$\partial\rho_{j1} = \frac{\begin{vmatrix} -dC_1 - \frac{\partial\rho_{j2}}{\partial\rho_{j1}}\, dC_2 - \frac{\partial\rho_{j3}}{\partial\rho_{j1}}\, dC_3 & C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j1}\partial\rho_{j2}} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j1}\partial\rho_{j3}} \\ -dC_2 - \frac{\partial\rho_{j1}}{\partial\rho_{j2}}\, dC_1 - \frac{\partial\rho_{j3}}{\partial\rho_{j2}}\, dC_3 & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j2}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j2}}^2} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j3}\partial\rho_{j2}} \\ -dC_3 - \frac{\partial\rho_{j1}}{\partial\rho_{j3}}\, dC_1 - \frac{\partial\rho_{j2}}{\partial\rho_{j3}}\, dC_2 & C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j3}\partial\rho_{j2}} & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j3}}^2} + C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j3}}^2} \end{vmatrix}}{\left|\underline{\underline{H}}\right|}$$

$$= \frac{-\left(dC_1 + \frac{\partial\rho_{j2}}{\partial\rho_{j1}}\, dC_2 + \frac{\partial\rho_{j3}}{\partial\rho_{j1}}\, dC_3\right)\begin{vmatrix} C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j2}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j2}}^2} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j3}\partial\rho_{j2}} \\ C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j3}\partial\rho_{j2}} & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j3}}^2} + C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j3}}^2} \end{vmatrix} + \left(dC_2 + \frac{\partial\rho_{j1}}{\partial\rho_{j2}}\, dC_1 + \frac{\partial\rho_{j3}}{\partial\rho_{j2}}\, dC_3\right)\begin{vmatrix} C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j1}\partial\rho_{j2}} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j1}\partial\rho_{j3}} \\ C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j3}\partial\rho_{j2}} & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j3}}^2} + C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j3}}^2} \end{vmatrix} - \left(dC_3 + \frac{\partial\rho_{j1}}{\partial\rho_{j3}}\, dC_1 + \frac{\partial\rho_{j2}}{\partial\rho_{j3}}\, dC_2\right)\begin{vmatrix} C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j1}\partial\rho_{j2}} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j1}\partial\rho_{j3}} \\ C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j2}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j2}}^2} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j3}\partial\rho_{j2}} \end{vmatrix}}{\left(C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j1}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j1}}^2}\right)\begin{vmatrix} C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j2}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j2}}^2} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j3}\partial\rho_{j2}} \\ C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j3}\partial\rho_{j2}} & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j3}}^2} + C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j3}}^2} \end{vmatrix} - \left(C_{j1}\frac{\partial\rho_{j1}}{\partial\rho_{j1}\partial\rho_{j2}}\right)\begin{vmatrix} C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j1}\partial\rho_{j2}} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j1}\partial\rho_{j3}} \\ C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j3}\partial\rho_{j2}} & C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j3}}^2} + C_{j2}\frac{\partial\rho_{j2}}{\partial{\rho_{j3}}^2} \end{vmatrix} + \left(C_{j1}\frac{\partial\rho_{j1}}{\partial\rho_{j1}\partial\rho_{j3}}\right)\begin{vmatrix} C_{j2}\frac{\partial\rho_{j2}}{\partial\rho_{j1}\partial\rho_{j2}} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j1}\partial\rho_{j3}} \\ C_{j1}\frac{\partial\rho_{j1}}{\partial{\rho_{j2}}^2} + C_{j3}\frac{\partial\rho_{j3}}{\partial{\rho_{j2}}^2} & C_{j3}\frac{\partial\rho_{j3}}{\partial\rho_{j3}\partial\rho_{j2}} \end{vmatrix}}$$

If no emissions intensities change, and the firm has no constraint of ideal emissions values, then there will be no change in all of these sourcing probabilities – the firm assumes it is already at the minimum and will simply pay the higher export costs. However, plugging in any nonzero value for any of the dC_j shows that $\frac{\partial{\rho_j}^*}{\partial C_j} \neq 0$ for the Cj in the country that is regulated.